Eur. Phys. J. B **41**, 97–105 (2004)
DOI: 10.1140/epjb/e2004-00299-1

THE EUROPEAN PHYSICAL JOURNAL B

Current and efficiency enhancement in Brownian motors driven by non Gaussian noises

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Received 19 March 2004 / Received in final form 8 July 2004
Published online 30 September 2004 –

Abstract. We study Brownian motors driven by colored non Gaussian noises, both in the overdamped regime and in the case with inertia, and analyze how the departure of the noise distribution from Gaussian behavior can affect its behavior. We analyze the problem from two alternative points of view: one oriented mainly to possible technological applications and the other more inspired in natural systems. In both cases we find an enhancement of current and efficiency due to the non-Gaussian character of the noise. We also discuss the possibility of observing an enhancement of the mass separation capability of the system when non-Gaussian noises are considered.

PACS. 05.45.-a Nonlinear dynamics and nonlinear dynamical systems – 05.40.Jc Brownian motion – 87.16.Uv Active transport processes; ion channels

1 Introduction

The study of noise induced transport by "ratchets" has attracted in recent years the attention of an increasing number of researchers due to the biological interest and also to its potential technological applications [1,2]. Since the pioneering works, besides the built-in ratchet-like bias and correlated fluctuations (see for instance [3]), different aspects have been studied, such as tilting [4,5] and pulsating [6] potentials, velocity inversions [4,7], etc. There are some relevant reviews [8,9] where the biological and/or technological motivation for the study of ratchets can be found.

Recent studies on the role of non Gaussian noises on some noise-induced phenomena like stochastic resonance, resonant trapping, and noise-induced transitions [10–15] have shown the possibility of strong effects on the system's response. For instance, enhancement of the signal-to-noise ratio in stochastic resonance, enhancement of the trapping current in resonant trapping, or shifts in the transition line for noise-induced transitions. These results motivate the interest in analyzing the effect of non Gaussian noises on the behavior of Brownian motors. Here we analyze the effect of a particular class of colored non Gaussian noise on the transport properties of Brownian motors. Such a noise source is based on the nonextensive statistics [16,17] with a probability distribution that depends on q, a parameter indicating the departure from Gaussian behavior: for $q = 1$ we have a Gaussian distribution, and different non Gaussian distributions for $q > 1$ or $q < 1$.

Some of the motivations for studying the effect of non Gaussian noises are, in addition to its intrinsic interest within the realm of noise induced phenomena, the existence of experimental data indicating that for several biological problems fluctuations have a non Gaussian character. Examples are current measurements through voltage-sensitive ion channels in a cell membrane or experiments on the sensory system of rat skin [18]. Also, recent detailed studies on the source of fluctuations in different biological systems [19] clearly show that, in such a context, noise sources are in general non Gaussian. Even though the previous arguments refer to biological aspects that are not directly related to ratchets, they strongly induce to think about the possible relevance of considering non Gaussian noises in those biological situations where the ratchet transport mechanism can play a role. In addition, from the point of view of technological applications, the finding of new conditions that may lead to an enhancement of the efficiency of the devices is always desirable. It is worth here remarking that there are some previous studies of non-Gaussian noise with similar tails [20]. For instance, those authors have analyzed aspects of the periodic attractors emerging from a saddle-node bifurcation plus noise that show chaos, or escape rates in noisy maps.

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We show here that, as a consequence of the non Gaussian character of the driving noise and from two alternative points of view, we can find a kind of enhancement of the system's response. The first –direct– point of view, following the line of previous works [10,12–15], takes as free parameters those that could be controlled in the case of technological applications. In this case we find a remarkable increase of the current together with an enhancement of the motor efficiency when non-Gaussian noises are considered (showing an optimum for a given degree of departure from the $q = 1$ Gaussian behavior). Moreover, when inertia is taken into account it is found that, again when departing from the Gaussian case, there seems to be a remarkable increment in the mass separation capability of these devices. The second point of view [11] is the more natural one when thinking of biological systems, as it considers the non-Gaussian noise as a primary source. In this case we also find an enhancement of the current and efficiency due to departure from Gaussian behavior, which occurs for relative low values of noise intensity. And the possibility of an inversion of current due only to changes on the non gaussianity of the noise.

Our analysis will include numerical simulations and analytical results coming from an adiabatic approximation valid for high correlation time of the forcing. We begin presenting the general framework within which we will work, and the nature of the non Gaussian noise. We continue discussing the first of the two points of view, and the results showing the enhancement we can find within it. After that we discuss the second point of view where we compare Gaussian and non Gaussian behaviors but adopting a constant width criterion, and discuss the results. Finally we draw some general conclusions.

2 Framework

We begin considering the general system

$$m\frac{d^2x}{dt^2} = -\gamma\frac{dx}{dt} - V'(x) - F + \xi(t) + \eta(t), \qquad (1)$$

where m is the mass of the particle, γ the friction constant, $V(x)$ the ratchet potential, F is a constant "load" force, and $\xi(t)$ the thermal noise satisfying $\langle\xi(t)\xi(t')\rangle = 2\gamma T\delta(t - t')$. Finally, $\eta(t)$ is the time correlated forcing (with zero mean) that allows the rectification of the motion, keeping the system out of thermal equilibrium even for $F = 0$. For this type of ratchet model several different kind of time correlated forcing have been considered in the literature [8,9]. In almost all studies authors have used Gaussian noises. The few exceptions which considered non Gaussian processes correspond mainly to the case of dichotomic noises [2,4,9].

The main characteristic introduced by the non Gaussian form of the forcing we consider here, is the appearance of arbitrary strong "kicks" with relatively high probability when compared, for example, with the Gaussian Ornstein–Uhlenbeck (OU) noise and, of course, with the dichotomic non Gaussian process.

2.1 Noise source

We will consider the dynamics of $\eta(t)$ as described by the following Langevin equation [10]

$$\frac{d\eta}{dt} = -\frac{1}{\tau}\frac{d}{d\eta}V_q(\eta) + \frac{1}{\tau}\zeta(t), \qquad (2)$$

with $\langle\zeta(t)\rangle = 0$ and $\langle\zeta(t)\zeta(t')\rangle = 2D\delta(t - t')$, and

$$V_q(\eta) = \frac{D}{\tau(q-1)} \ln\left[1 + \frac{\tau}{D}(q-1)\frac{\eta^2}{2}\right].$$

Previous studies of such processes in connection with stochastic resonance problems [10,11] and dynamical trapping [13], have shown that the non Gaussian behavior of the noise leads to remarkable effects. For $q = 1$, the process η coincides with the OU one (with a correlation time equal to τ), while for $q \neq 1$ it is a non Gaussian process. As shown in [10], for $q < 1$ the stationary probability distribution has a bounded support, with a cut-off at $|\eta| = \omega \equiv [(1-q)\tau/(2D)]^{-\frac{1}{2}}$, with a form given by

$$P_q(\eta) = \frac{1}{Z_q}\left[1 - \left(\frac{\eta}{\omega}\right)^2\right]^{\frac{1}{1-q}}, \qquad (3)$$

for $|\eta| < \omega$ and zero for $|\eta| > \omega$ (Z_q is a normalization constant). Within the range $1 < q < 3$, the probability distribution is given by

$$P_q(\eta) = \frac{1}{Z_q}\left[1 + \frac{\tau(q-1)\eta^2}{2D}\right]^{\frac{1}{1-q}} \qquad (4)$$

for $-\infty < \eta < \infty$, and decays as a power law (slower than a Gaussian distribution). Finally, for $q > 3$, this distribution can not be normalized.

Hence, we see that keeping D constant, the width or dispersion of the distribution increases with q. This means that, the higher the q, the stronger the "kicks" that the particle will receive. Figure 1 depicts the typical form of this distribution for q smaller, equal and larger than 1.

In [10] it was shown that the second moment of the distribution, which we will interpret as the "intensity" of the non Gaussian noise, is given by

$$D_{ng} \equiv \langle\eta^2\rangle = \frac{2D}{\tau(5-3q)}, \qquad (5)$$

which diverges for $q \geq 5/3$. For the correlation time τ_{ng} of the process $\eta(t)$, defined in detail in [10] it is not possible to find an analytical expression. However, it is known [10] that for $q \to 5/3$ it diverges as $\sim (5-3q)^{-1}$. In our analysis, we will consider values of q in the range $0.5 < q < 5/3 \simeq 1.66$. For this interval we have studied numerically the dependence of τ_{ng} on q, and we have found the following analytical approximation

$$\tau_{ng} \simeq 2\,\frac{\left[1 + 4(q-1)^2\right]\tau}{(5-3q)}, \qquad (6)$$

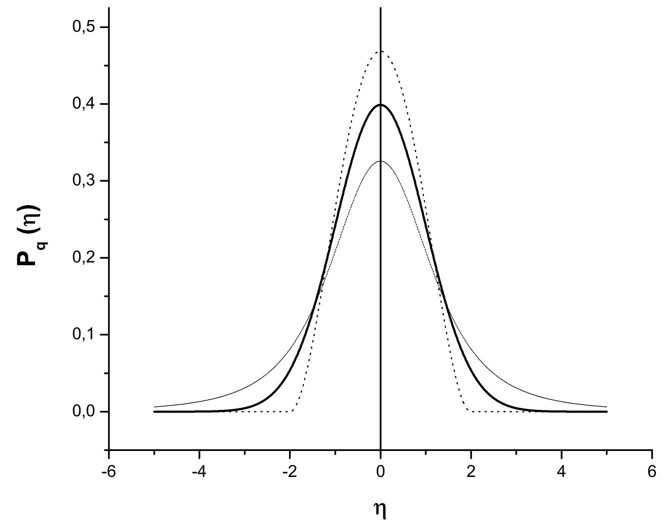

Fig. 1. PDF vs. η for different values of q, and for $D/\tau = 1$. From top to bottom, the dotted line is for $q = 0.5$, the thick continuous line is for $q = 1$ (Gaussian case), and the thin continuous line is for $q = 1.5$.

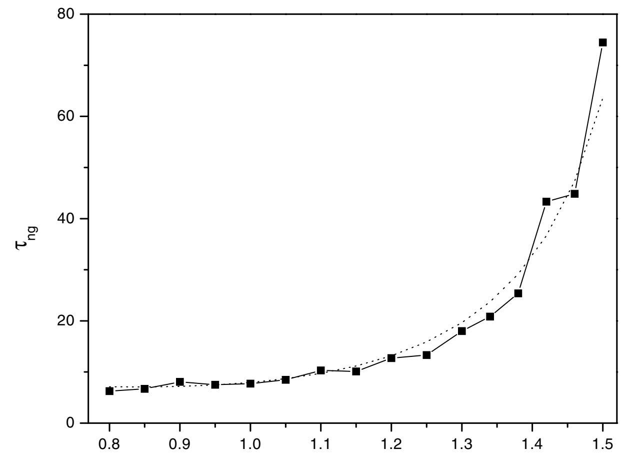

Fig. 2. Comparison of simulations (line with squares) and the analytical approximation (dotted line) for τ_{ng} vs. q.

that fits very accurately the results. In Figure 2 we compare the results from simulations for the correlation time as a function of q with the formula on equation (6). This fitting will be the one we will consider in Section 4, when analyzing the dependence of the transport properties of the ratchet system on the intrinsic parameters of the non Gaussian noise, D_{ng} and τ_{ng}.

2.2 System parameters

Here we point out the way in which we will explore the parameter space in order to find relevant and general phenomena. The system parameters appearing in equations (1) and (2), are m, γ, F, T, D, τ and q. First, it is possible to scale out the parameter γ, and set $\gamma = 1$. We will do this in most calculations, except in some cases where we will consider $\gamma = 2$ in order to get to the parameter region considered in a previous work in the clearest possible way.

It is clear that exploring the whole parameter space is an extremely tedious (and probably uninteresting) task. Instead, we will focus in some parameter region where interesting phenomena have been reported for ratchets with Gaussian noises, and change the characteristics of the forcing (basically, the parameter q) in order to analyze the effects of the non gaussianity.

Concerning the parameters characterizing the non Gaussian noise, we will analyze the changes in the system behavior when varying q at constant D and τ first, and then at constant D_{ng} and τ_{ng}. The general behavior of most of the noise induced phenomena in terms of the noise intensity is the appearance of an optimum intensity for which the response of the system is maximum. This is because small noise intensities are generally not enough to produce notable effects (specially when the system has threshold characteristics), and very large noise intensities leads to a dynamics which is completely dominated by noise and other characteristics of the system are lost. In this work we will study the behavior of the system for values of noise intensities (D or D_{ng}) that goes from zero to values where the noise induced phenomena (directed transport) is well developed. We will not analyze the large noise intensity regime in which the transport start to decrease. For the correlation time of the forcing (τ or τ_{ng}) we will consider only relatively high values, in order to compare with the adiabatic theory existing for $\tau \to \infty$. The low correlation time regime, which may be also an interesting feature, will remain unexplored.

Concerning the temperature we will consider low values —in order to have the non Gaussian forcing as the main source of noise— but non zero in most cases, in order to develop the adiabatic theory. For F we will consider values comparable (but lower) to those of $V'(x)$. When studying inertial effects we will consider ranges of m similar to those appearing in the literature in order to compare results.

As stated above, the parameter q characterizes the non Gaussian properties of the noise distribution ($q = 1$ corresponding to the Gaussian case). We will consider values of q ranging from $q \sim 0.6$ to $q \sim 5/3 \simeq 1.66$, that is, surrounding the Gaussian case, and where some interesting results are found for $q > 1$. For $q < 0.6$ we expect the transport phenomena to be reduced, as the noise distribution is bounded for $q < 1$ and the bounds decreases with decreasing q. For $q \to 5/3$ the efficiency goes to zero because of the divergence of the second moment of the noise distribution. Hence, we do not consider higher values of q.

3 Analysis for constant D and τ

As our first approach we will analyze the results for the current and efficiency as function of q for constant values of D and τ. These are the parameters that, together with q, could be primary controlled in a designed technological device. For instance, it is possible to think on

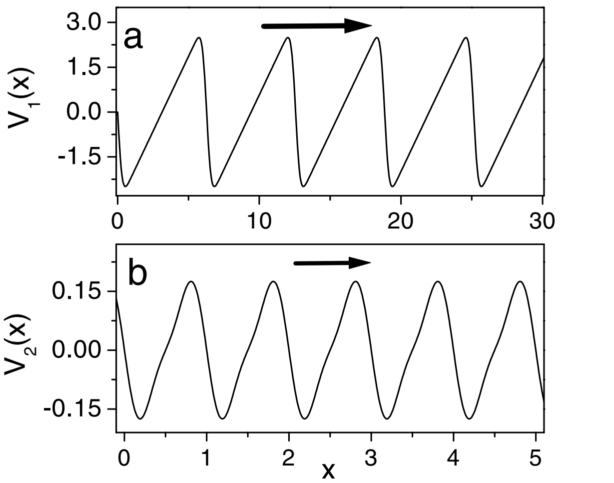

Fig. 3. The ratchet potentials considered in this work: $V_1(x)$ (a) and $V_2(x)$ (b). The arrows indicate the direction of the current in normal situations (that is, when there is no current "inversion").

an electronic circuit whose output is governed by equation (2), where τ and D are determined by adequately tuning the values of capacitors, inductances, resistances or other system's components [22]. By performing this analysis we are continuing (and complementing) the line of previous works [10,12–15] which studied the response of different systems to a noise source given by equation (2) considering D, τ and q as the control parameters.

3.1 Overdamped system

Firstly, we analyze the overdamped regime setting $m = 0$ and $\gamma = 1$. For the ratchet potential in this case we will consider the same form as in [3] (with period $L = 2\pi$)

$$V(x) = V_1(x) = -\int^x dx' \left(\frac{\exp[\alpha \cos(x')]}{J_0(i\alpha)} - 1 \right), \quad (7)$$

where $J_0(i\alpha)$ is the Bessel function, and $\alpha = 16$. The form of $V_1(x)$ is shown in Figure 3a. The integrand in equation (7) is the ratchet force $(-V')$ appearing in equation (1).

We are interested on analyzing the dependence of the mean current $J = \langle \frac{dx}{dt} \rangle / L$ and the efficiency ε on the different parameters, in particular, their dependence on the parameter q. The efficiency is defined as the ratio of the work (per unit time) done by the particle "against" the load force F

$$\lim_{T_f \to \infty} \frac{1}{T_f} \int_{x=x(0)}^{x=x(T_f)} F dx(t),$$

to the mean power injected to the system through the external forcing η

$$\lim_{T_f \to \infty} \frac{1}{T_f} \int_{x=x(0)}^{x=x(T_f)} \eta(t) dx(t).$$

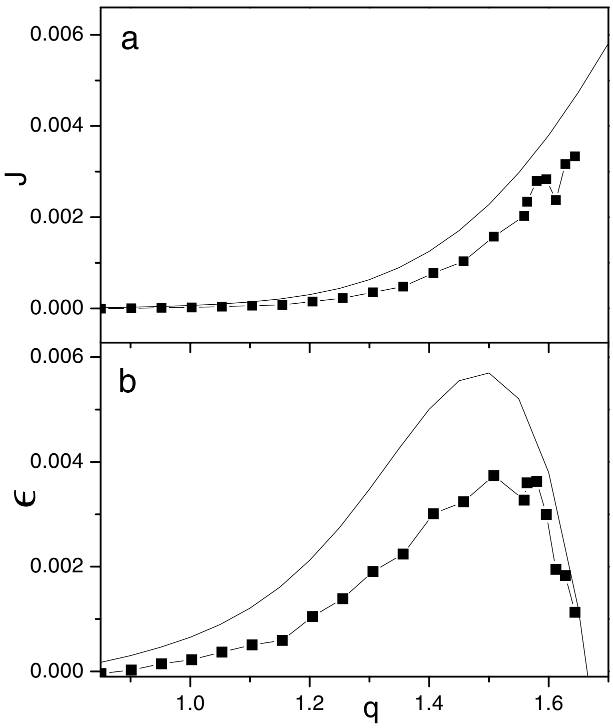

Fig. 4. Current (a) and efficiency (b) as functions of q. The solid line corresponds to the adiabatic approximation, the line with squares shows results from simulations. All calculations are for $m = 0, \gamma = 1, kT = 0.5, F = 0.1, D = 1$ and $\tau = 100/(2\pi)$.

For the numerator we get $F\langle \frac{dx}{dt} \rangle = FJL$, while for the denominator we obtain

$$\lim_{T_f \to \infty} \frac{1}{T_f} \int_0^{T_f} \eta(t) \frac{dx}{dt} dt = \frac{1}{\gamma} (\langle -V' \eta \rangle + \langle \eta^2 \rangle).$$

Simulations show that the time average of $V'(x(t))\eta(t)$ is negligible in the latter equality (it is always several orders of magnitude lower than $\langle \eta^2 \rangle$) and we may approximate the denominator as $\langle \eta^2 \rangle / \gamma = 2D[\gamma\tau(5-3q)]^{-1}$. Interesting and complete discussions on the thermodynamics and energetics of ratchet systems can be found in [21].

In the overdamped regime we are able to give an approximate analytical solution for the problem, which is expected to be valid in the large correlation time regime ($\frac{\tau}{D} \gg 1$): we perform the adiabatic approximation of solving the Fokker-Planck equation associated to equation (1) assuming a constant value of η [23], analogous to the one used in [24]. This leads us to obtain an η–dependent value of the current $J(\eta)$ that is then averaged over η using the distribution $P_q(\eta)$ [10] with the desired values of q, D and τ

$$J = \int d\eta \, J(\eta) \, P_q(\eta).$$

We remark that, although the Fokker-Plank equation is solved in the $\tau/D \to \infty$ limit, the solution we find depends on D and τ through the $P_q(\eta)$ distribution.

In Figure 4, we show typical analytical results for the current and the efficiency as functions of q together with

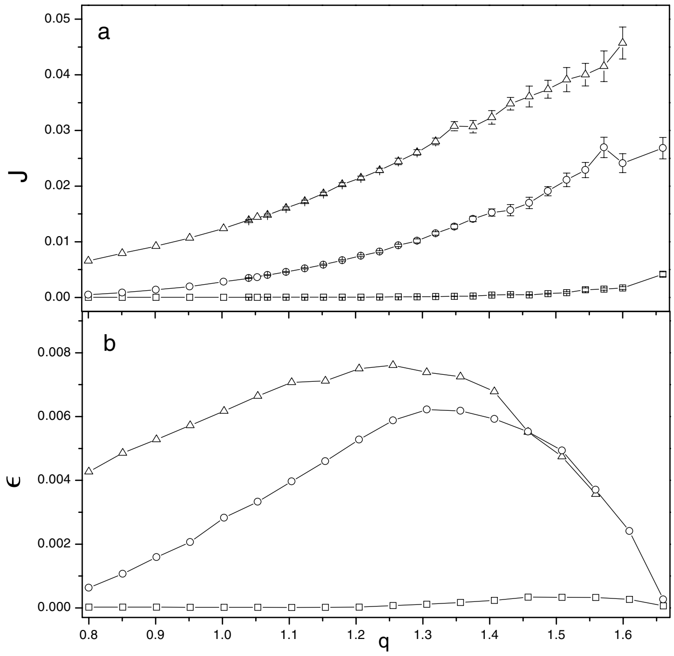

Fig. 5. Current (a) and efficiency (b) as functions of q. Results from simulations at $T = 0$ for $D = 1$ (squares), $D = 10$ (circles), and $D = 20$ (triangles). All calculations are for $m = 0, \gamma = 1, F = 0.1$ and $\tau = 100/(2\pi)$.

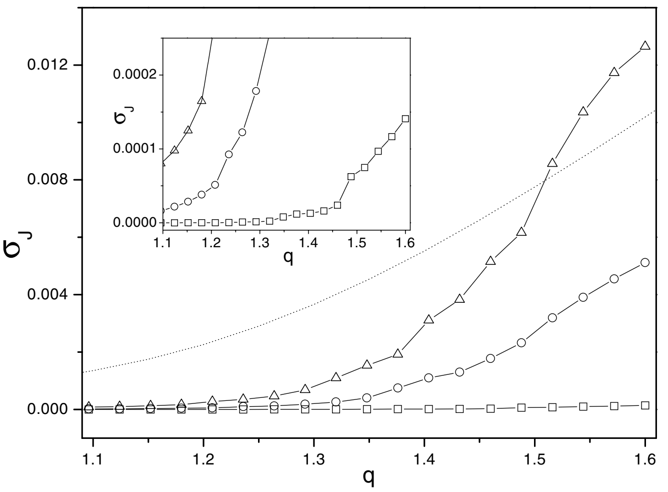

Fig. 6. Variance of J as function of q. The lines with symbols corresponds to results from simulations at $T = 0$ for: $D = 1$ (squares), $D = 10$ (circles), and $D = 20$ (triangles). The dotted line without symbols corresponds to the adiabatic theory (at a *non zero* temperature $T = 0.25$) for $D = 10$. The rest of the parameters are the same as in Figure 5. The inset shows the same curves with a different vertical scale that makes more apparent the growth of σ_J for $D = 1$ (line with squares).

results coming from numerical simulations (for the complete system given by Eqs. (1) and (2)). Calculations have been done in a parameter region similar to that studied in [3] but considering (apart from the difference provided by the non Gaussian noise) a non–zero load force that leads to a non–vanishing efficiency. As can be seen, although there is not a quantitative agreement between theory and simulations, the adiabatic approximation predicts qualitatively very well the behavior of J and ε as q is varied. As shown in the figure, the current grows monotonously with q (at least for $q < 5/3$) while there is an optimal value of q (> 1) which gives the maximum efficiency. This fact could be interpreted as follows: when q is increased, the width of the $P_q(\eta)$ distribution grows and high values of the non Gaussian noise become more frequent, leading to an improvement of the current. Although the mean value of J increases monotonously with q, the efficiency decays when q approaches $5/3$ since the denominator in the definition of ε, which is essentially the dispersion of the noise distribution, diverges. As we will shown, the decay of the efficiency is also associated to an increase on the fluctuations of J.

In Figure 5 we show results from simulations for J and ε as functions of q for different values of D, the intensity of the white noise in equation (2). The results correspond to $T = 0$, hence, the only noise present in the system is the non Gaussian one. The behavior of J and ε as function of q is essentially the same appearing in Figure 4. In Figure 6 we show σ_J, the variance of the current, as a function of q, from the same simulations as in Figure 5. It can be seen that the decay of the efficiency occurs in the range of q where σ_J exhibits a huge growth, reaching values of the order of J^2 and even larger. This lead us to think on a connection between the decay of efficiency and the growth of the fluctuations of J to values of the order of J itself. A simple and intuitive interpretation of such connection can be given: for $q \to 5/3$, in spite of having a large (positive) mean value of the current, for a given realization of the process the transport of the particle towards the desired direction is far from being assured (due to the large fluctuations on J). Hence, the transport mechanism ceases to be efficient.

We note that, although it is easier to interpret the decay of ε with q as being a consequence of the increase of σ_J instead of to the increase of D_{ng}, from the point of view of the definition of the efficiency, the decay is due to the latter cause. However, the adiabatic theory allow us to link the growth of D_{ng} with that of σ_J and view both phenomena as equally connected to the decay of the efficiency. In Figure 6 we have also shown results for σ_J as a function of q coming from the adiabatic theory. (σ_J is computed as $\langle J(\eta)^2\rangle - \langle J(\eta)\rangle^2$ with $J(\eta)$ obtained from the Fokker–Plank equation for $\tau \to \infty$ and the averages taken with respect to the stationary $P_q(\eta)$ distribution.) It can be observed that the theory qualitatively agrees with simulations, as it predicts a monotonously increase of σ_J with q which is in the same scale of the one computed from simulations (it should be noted, that the adiabatic theory is developed at a non–zero temperature).

The monotonous increase of both σ_J and D_{ng} with q (for constant D and τ) indicates that, in principle, it is possible to obtain an analytical one–to–one relation between both magnitudes. The actual computations are rather tedious since involve inverting integral relations that we have solved numerically.

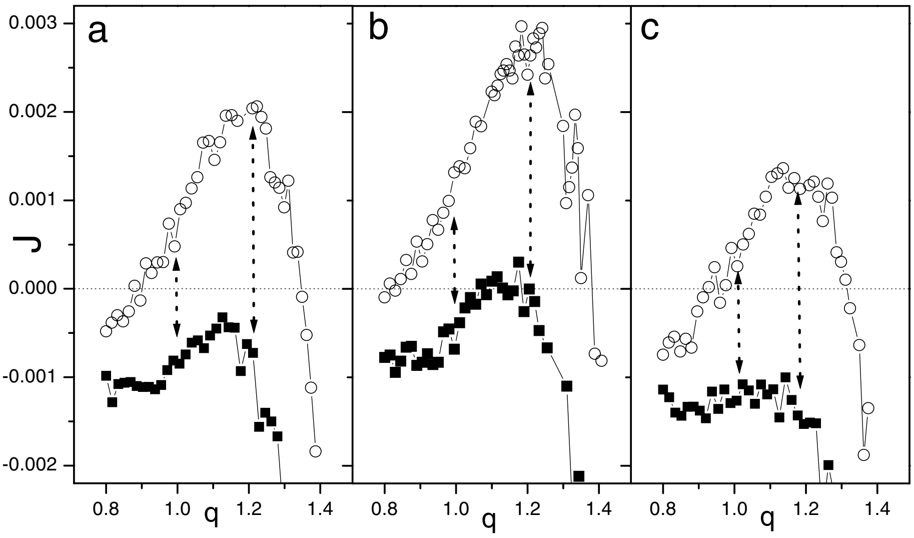

Fig. 7. Separation of masses: results from simulations for the current as a function of q for particles of masses $m = 0.5$ (hollow circles) and $m = 1.5$ (solid squares). Calculations for three different values of the load force: $F = 0.025$ (a), $F = 0.02$ (b) and $F = 0.03$ (c).

It is interesting to mention another phenomenon that could be related to the decay of the efficiency —although we will not delve deep into this point— which is the divergence of the "flatness" of the noise distribution. The flatness of a noise distribution is defined as the ratio of the fourth moment to the square of the second moment [4]. It is interesting to note that the flatness of $P_q(\eta)$ is infinite for $q > 7/5 = 1.4$, which is approximately the parameter region where we observe the decay of the efficiency. In [4], the dependence of the current on the flatness of the noise distribution was discussed for a ratchet system similar to the one here studied, however, it was done in the limit of small correlation time and considering a different kind of random forcing.

It is worth recalling here that $q = 1$ corresponds to the Gaussian OU noise [9]. Hence, the results for constant D and τ show that the transport mechanism becomes more efficient when the stochastic forcing has a non Gaussian distribution with $q > 1$. Let us now discuss the case with inertia.

3.2 Inertial system

Now we turn to study the $m \neq 0$ case, that is, the situations in which the inertia effects are relevant. Hence, we consider the complete form of equation (1).

We have studied the dependence of the current J on the mass m for different values of q at constant D and τ, finding that the results (not shown) are in general similar to those appearing in [25,26] for $q = 1$. As m is increased from 0, the inertial effects initially contribute to increase the current, until an optimal value of m is reached. As expected, for high values of m, the motion of the particle becomes more difficult and, for $m \to \infty$, the current vanishes. As stated in [25,26], eventually (depending on the parameters) an inversion of the current is observed for a well defined range of values of m. In such situations, the phenomenon of mass separation becomes possible, as particles with different masses moves in opposite directions. For our system, the region of parameters where this phenomenon occurs may now depend on q and differs from those in [25,26] corresponding to $q = 1$.

In order to study specifically the influence of the non gausiannity of the noise in the phenomenon of mass separation, we analyze here the same system studied in [25,26] but considering the non Gaussian forcing described by equation (2). We fix

$$V(x) = V_2(x) = -\frac{2}{\pi}[\sin(2\pi x) + 0.25\sin(4\pi x)]$$

in equation (1) as the ratchet potential, which is shown in Figure 3b. We focus on the region of parameters where, in [25] (for $q = 1$), separation of masses was found. We fix $\gamma = 2, T = 0.1, \tau = 0.75$, and $D = 0.1875$ and assume the values of the masses $m = m_1 = 0.5$ and $m = m_2 = 1.5$ as in [25]. Our main result is that the separation of masses is enhanced when a non–Gaussian noise with $q > 1$ is considered. In Figure 7a we show J as function of q for $m_1 = 0.5$ and $m_2 = 1.5$. It can be seen that there is an optimum value of q that maximizes the difference of currents. This value, which is close to $q = 1.25$, is indicated with a vertical double arrow. Another double arrow indicates the separation of masses occurring for $q = 1$ (Gaussian OU forcing). We have observed that when the value of the load force is varied, the difference between the curves remains approximately constant but both are shifted together to positive or negative values (depending on the sign of the variation of the loading). By controlling this parameter it is possible to achieve, for example, the situation shown in Figure 7b, where, for the value of q at which the difference of currents is maximal, the heavy "species" remains static on average (has $J = 0$), while the light one has $J > 0$. It is also possible to get the situation shown in Figure 7c, at which, for the optimal q, the two species move in opposite directions with equal average velocity.

4 Analysis for constant D_{ng} and τ_{ng}

We consider here the second point of view. It corresponds to studying the behavior of current and efficiency as function of q when D_{ng} and τ_{ng} are kept constant (instead of D and τ as was done in the previous section). This approach is more consistent with the image of η as a primary (natural) source of noise, hence it can be considered more relevant from the point of view of biological systems. Moreover, it isolates the effects of the non-Gaussian character of the noise distributions by keeping the dispersion and correlation time at fixed values.

4.1 Overdamped case

We consider equation (1) with $m = 0$ and $V(x)$ as in Figure 3a (see Sect. 3.1, overdamped regime). In the simulations, for each value of q, we have adapted the values of D and τ in order to obtain the desired values of D_{ng} and τ_{ng}. This was done by inverting equations (5) and (6).

As q is varied for constant D_{ng}, the efficiency is essentially proportional to the current and the curves for $\varepsilon(q)$ have the same behavior than those for $J(q)$. Hence, we only present results for J. In Figure 8 we show the results from both, the adiabatic theory and the numerical simulations, for the current as function of q for $\tau_{ng} = 2\pi/100$ and different values of D_{ng}. An interesting result is found for low values of D_{ng}. For $D_{ng} < 0.5$ it is observed that the current grows monotonously with q through most of the range studied. For q very close to 5/3 it decays, since $D \to 0$. This means that, for $q > 1$ we found an enhancement of J with respect to the Gaussian noise situation ($q = 1$) with an optimum value of $q < 5/3$. This enhancement has to be attributed essentially to the non-Gaussian character of the noise $\eta(t)$, as for every q, on each curve, we have considered the same values of dispersion and correlation time.

For higher values of D_{ng} the effect disappears: for ($D_{ng} \sim 1$) the dependence of J on q becomes flat in most of the range analyzed. This means that the non-Gaussian character does not play a relevant role in this region of parameters, and the current and efficiency are essentially determined by the intensity and correlation time of the noise source η, independently of the detailed statistical characteristics of the process $\eta(t)$. For even higher values of $D_{ng}(\sim 2)$ the optimum value of q that maximizes the current tends toward values of $q < 1$ (results not shown). However, the curves are essentially flat, the differences with the $q = 1$ case being not remarkable at all. Hence, we can remark that the enhancement effect occurs for $q > 1$ at low values of D_{ng}.

4.2 Inertial systems

We consider now the case $m \neq 0$, and study the current as a function of q for fixed values of D_{ng} and τ_{ng}. We fix $\gamma = 1$ and consider again the potential $V(x) = V_2(x)$ defined in Section 3.2.

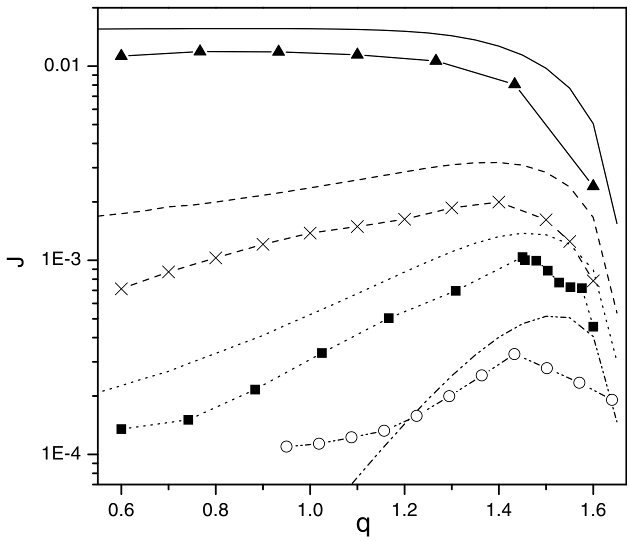

Fig. 8. Current as a function of q for fixed $\tau_{ng} = 100/(2\pi)$ and different fixed values of D_{ng}. The lines with symbols corresponds to simulations and the lines without symbols to the adiabatic theory. From top to bottom, the curves are for $D_{ng} = 1$ (solid line for theory and solid line with triangles for simulations); $D_{ng} = 0.35$ (dashed line and dashed line with crosses); $D_{ng} = 0.2$ (dotted line and dotted line with squares); and $D_{ng} = 0.1$ (dash-dot-dot line and dash-dot-dot line with circles). All calculations are for $m = 0, \gamma = 1, T = 0.5$ and $F = 0.1$.

In Figure 9 we show the results for $J(q)$ at fixed $D_{ng}(= 0.1875)$ and $\tau_{ng}(= 0.75)$, for different values of the masses and the external force F. It can be seen that, when considering $q \neq 1$, no remarkable enhancement of the mass separation capabilities of the system is found. Moreover, for $q > 1$ the mass separation effect decreases. However, an interesting phenomenon occurring is an inversion of current when considering large enough values of q (depending on the mass and F). This inversion of current is due essentially to the variation of the non-Gaussian properties of the noise distribution, since the dispersion D_{ng} and correlation time τ_{ng} are kept fixed.

We want to stress that D_{ng} and τ_{ng} are the relevant parameters of the non Gaussian noise source when we think of such a noise as the "primary source" acting on the Brownian particle. Hence, the results shown in this Section for the current as a function of q for constant values of D_{ng} and τ_{ng}, contributes to isolate the effect of the non Gaussian character of the noise. In this way, from the results for "equivalent" Gaussian and non Gaussian noises, we observe that non Gaussian noises with $q > 1$ produce an enhancement of the current when compared to the "equivalent" Gaussian case.

5 Conclusions

We have systematically studied the effect of a colored non Gaussian noise source on the transport properties of a

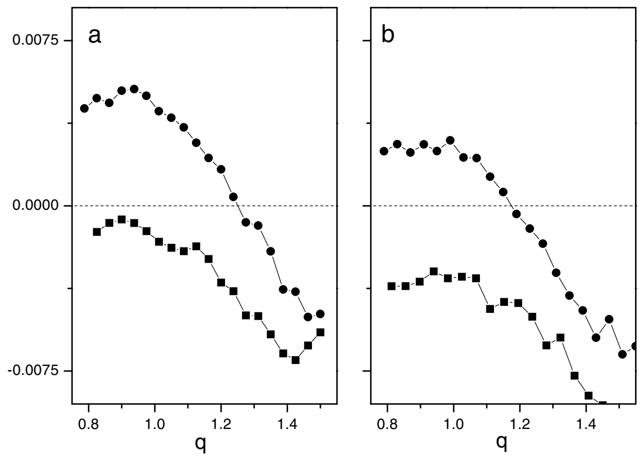

Fig. 9. Current as a function of q for fixed $\tau_{ng} = 0.75$ and $D_{ng} = 0.1875$. Figure (a) is for $F = 0.015$ and (b) is for $F = 0.025$. In both cases circles correspond to $m = 0.5$ while squares correspond to $m = 1.5$.

Brownian motor using two alternative points of view. In the first one, we analyze the results for the current, efficiency and mass separation as functions of q, for constant values of D and τ, which are the parameters that could be adequately controlled, for example, in a designed technological device. The second point of view corresponds to studying those behaviors as functions of q when D_{ng} and τ_{ng} are kept constant. This is more consistent with the image of η as a primary (natural) source of noise, and it isolates the effects of the non-Gaussian character of the noise distributions by keeping the dispersion and correlation time at fixed values.

Considering the first, direct, point of view, we have found that a departure from Gaussian behavior, in particular given by a value of $q > 1$, induces a remarkable increase of the current together with an enhancement of the motor efficiency. The latter shows, in addition, an optimum value for a given degree of non gaussiannity and decays due to the enhancement of fluctuations when the correlation of the non Gaussian noise diverges. When inertia is taken into account we have also found a considerable increment in the mass separation capability of the system.

The second point of view is analogous to the one used in reference [11] to study stochastic resonance in an activator-inhibitor system, where it was shown that the signal-to-noise ratio shows an enhancement as a function of q. Here, by keeping the distribution's width D_{ng} and the correlation time τ_{ng} constant, we have compared the results for "equivalent" Gaussian and non Gaussian noises. We have observed that non Gaussian noises with $q > 1$ produce an enhancement of the current when compared to the Gaussian case. This effect is observed for relatively low values of the noise intensity D_{ng} and lead us to interpret that, at low values of D_{ng}, the increment of the probability of having arbitrary high values of the noise that occurs for $q > 1$ (with respect to the Gaussian case) plays a significative role in the determination of the current. In contrast, for higher values of D_{ng}, the fluctuations dominate the dynamics in such a way that the Gaussian or non Gaussian character of the noise produces no relevant differences.

When studying inertial systems at constant D_{ng} and τ_{ng} we have not observed relevant effects on the mass separation capability of the system, in the region of parameters considered: the effect of mass separation is governed essentially by the noise intensity and the correlation time.

Another remarkable fact is the occurrence of an inversion of current as a consequence of varying the parameter q alone (keeping D_{ng} and τ_{ng} fixed). This clearly shows the relevance that the details of the noise distribution may have in the determination of the transport properties in ratchet systems or, equivalently, how sensitive the ratchet systems could be to the detailed properties of the noises.

We think that these studies could be of interest for their possible relation to biologically motivated problems [1,18,19] as well as for the potential technological applications, for instance in "nanomechanics" [8,9]. More specific studies on these areas will be the subject of further work.

The authors thank V. Grunfeld for a revision of the manuscript. Partial support from ANPCyT, Argentine agency, is acknowledged. HSW wants to thank to the European Commission for the award of a *"Marie Curie Chair"*, and to the IFCA and Universidad de Cantabria, Santander, Spain, for the kind hospitality extended to him.

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